Gerry ő

Final exam for Kwantumfysica 1 - 2006-2007

Thursday 28 June 2007, 9:00 - 12:00

READ THIS FIRST:

- Clearly write your name and study number on each answer sheet that you use.
- On the first answer sheet, write clearly the total number of answer sheets that you turn in.
- Note that this exam has 3 questions, it continues on the backside of the papers!
- Start each question (number 1, 2, 3) on a new answer sheet.
- The exam is open book within limits. You are allowed to use the book by Liboff, and one A4 sheet with notes, but nothing more than this.
- If it says "make a rough estimate", there is no need to make a detailed calculation, and making a simple estimate is good enough. If it says "calculate" or "derive", you are supposed to present a full analytical calculation.
- If you get stuck on some part of a problem for a long time, it may be wise to skip it and try the next part of a problem first.

Useful formulas and constants:

Electron mass	m_e	$= 9.1 \cdot 10^{-31}$ kg
Electron charge	$-e$	$= -1.6 \cdot 10^{-19}$ C
Planck's constant	h	$= 6.626 \cdot 10^{-34}$ Js $= 4.136 \cdot 10^{-15}$ eVs
Planck's reduced constant	$\hbar$	$= 1.055 \cdot 10^{-34}$ Js $= 6.582 \cdot 10^{-16}$ eVs

Fourier relation between x-representation and k-representation of a state

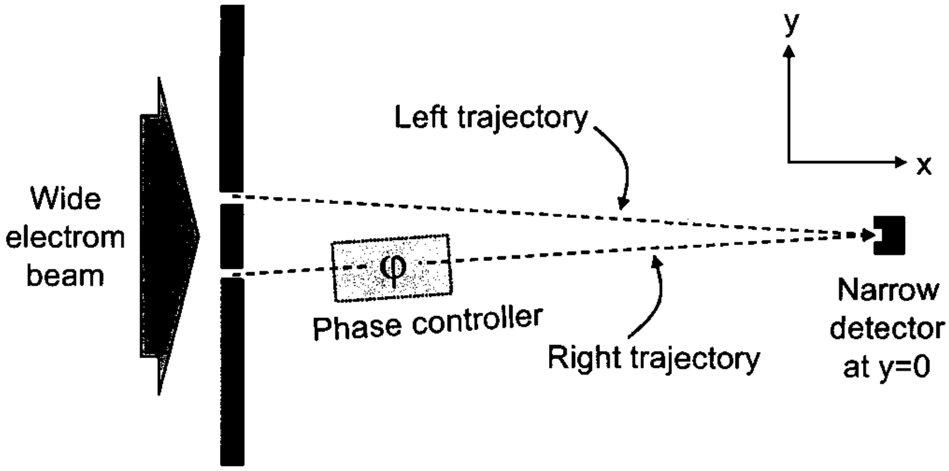
$$\Psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \overline{\Psi}(k) e^{ikx}\, dk$$

$$\overline{\Psi}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Psi(x) e^{-ikx}\, dx$$

Z.O.Z.

Problem 1 - USE DIRAC NOTATION FOR THIS PROBLEM

Consider the double slit experiment in the figure, that is suited for demonstrating quantum interference of electrons. A very stable and wide electron beam (diameter of the Gaussian profile is much larger than distance between the slits) is incident on a thin metal screen with two slits. The electrons in the beam are accelerated and arrive with a kinetic energy of 5000 eV at the screen. The screen is at $x = 0$ m. The slits are at $y = -100$ μm and $y = +100$ μm, and have a width of 1 μm. An electron detector (counter) is placed far from the screen at position $x = 1$ m, $y = 0$ m.



The electrons that pass the screen are in a quantum superposition of states that result from coming out of the left slit, and coming of the right slit. Part of these electrons are in a state that is directed towards the detector opening. For the left trajectory, this state is denoted as the state $|\Psi_L\rangle$. For the right trajectory this state is denoted as the state $|\Psi_R\rangle$. At the entrance of the detector, these two states overlap again nearly perfectly, such that there $\langle\Psi_L|\Psi_R\rangle = 1$.

In the path of one of the slit-detector trajectories a phase controller is placed. The phase (φ) controller is formed by two metal plates that lie parallel to the x-y plane, closely above and under the beam. The length in x-direction is L. By setting the same voltage V_φ (with respect to the screen) on the two plates you can control the potential energy $-eV_\varphi$ that the electrons experience between the plates, and thereby influence the phase difference between electron states in the left and right trajectory of the setup.

a) Calculate the de Broglie wavelength of the electrons that pass the screen (but before they enter the phase controller).

b) Derive an expression (use E_{k0} as notation for the electron kinetic energy before entering the phase controller) for the de Broglie wavelength of the electrons while they are inside the phase controller, with the voltage set at a value $V_\varphi < 0$ V, but with $-eV_\varphi < E_{k0}$.

c) Assume that the phase controller can be operated such that it only influences the phase of the electron states (that is, no back reflections of electrons on the phase controller, or reflections to the sides). If the phase controller is off (set to $V_\varphi = 0$), the count rate r of detector is $r_0 = 1000$ electrons per second. Derive an expression for how r depends on φ, and make a graph of r as a function of φ for $0 < \varphi < 8\pi$.

d) Repeat question **c)**, but now consider the more realistic situation that due to reflections (or scattering) on the phase controller, the transmission coefficient of the entire phase controller is only $T = 0.64$ (and independent of φ).

e) Calculate r for the case that the right slit is closed (blocking all electrons for that slit, only the left slit open, all other conditions are kept the same).

f) Assume now that the transmission of the phase controller can be restored to full transmission ($T = 1$, that is, assume again the conditions as for question **c)**). However, now there is another complication with the setup. There is noise on the control voltage V_φ. As a result, setting the phase controller at φ, results in reality in the situation that for electrons that are transmitted through the phase controller, the phase φ is in fact $\varphi \pm \Delta\varphi$. Here $\Delta\varphi$ is an uncertainty of 5% that takes on a random value, different for each electron (assume a uniform distribution). Calculate what now the count rate r is for the case that one aims at setting up $\varphi = 2\pi$.

g) Assume again the situation of question **f)**, and make for this situation a graph of r as a function of φ for $0 < \varphi < 8\pi$. If you do not have an answer for question **f)**, try to give a qualitative sketch, and explain your answer.

Z.O.Z.

Problem 2

Consider the following model system for an atom with one electron: A one-dimensional particle-in-a-box system, where the potential for the electron outside the box is infinite, and inside the box the potential $V = 0$. The position of the electron is described by a coordinate x. The width of the box is a, with the walls at $x = -a/2$ and $x = +a/2$.

a) Derive the four energy eigenvalues with the lowest energy, and describe the corresponding energy eigenstates $\varphi_n(x)$.

b) Assume that this system has an electrical dipole moment that oscillates when the system is emitting a photon. This can occur when the system is in a superposition of two different energy eigenstates $|\varphi_m\rangle$ and $|\varphi_n\rangle$. The operator for this dipole moment is $\hat{D} = e\hat{X}$, where $\hat{X}$ the position operator and e the electron charge. Use symmetry arguments to show that this system cannot emit a photon when it is in a state that is a superposition of two energy eigenstates with the same parity. Hint: use the x-representation to evaluate $\langle\varphi_n|\hat{D}|\varphi_m\rangle$.

c) An electron is in the third excited state of this system (from the four lowest energy eigenstates, the one with the highest energy). It can (and will) relax to lower energy eigenstates by spontaneous emission of a photon during the transition to this lower state. Discuss which relaxation processes are possible, and for each which photon is (or photons are) emitted, and what the final state is.

Problem 3 - USE x- and k-representation FOR THIS PROBLEM

Consider a free particle in one dimension (position x) with mass m. At some time $t = 0$, the (normalized) state of the particle in x-representation is

$$\Psi(x, t=0) = \frac{1}{\sqrt{a}} e^{\frac{-|x|}{a}} \qquad (1).$$

a) Give an expression for the Hamiltonian of this system.

b) What is the time-evolution operator $\hat{U}$ for this system, in terms of position and/or momentum operators?

c) Give an expression for the state of an electron that behaves as a plane wave moving in the positive x-direction (also a particle in one-dimension).

d) Give an expression that describes the time evolution of state (1) in terms of plane waves, such that you can describe the state at an arbitrary time $t > 0$. Explain why it is useful or not useful to use a Fourier transformation between representations